TTP00-18
hep-ph/0008180
August 2000

Kinematic Effects in Quarkonia Production *

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Abstract

We investigate energy (momentum) distributions in J/ψ photoproduction and J/ψ production in B meson decay. In particular the upper endpoint region of the spectrum is examined where the effect of soft gluon emission from the $c\bar{c}$ pair becomes important. Constructing a model which is consistent with the so-called shape function formalism we consider these fragmentation effects and show that the relevance of possible colour octet contributions in the photoproduction channel is still inconclusive.

*Talk given at the conference "Hyperons, Charm and Beauty Hadrons", Valencia, June 2000. To be published in Nucl. Phys. Suppl.

1 Introduction

Non-relativistic QCD (NRQCD) [1] provides the framework for a successful description of many heavy quarkonia production processes. Performing a systematic expansion in the typical velocity v of a (anti)quark inside the quarkonium NRQCD includes modes where the quark antiquark pair ($Q\overline{Q}$ pair) is produced in a colour octet state and hadronize to a colourless quarkonium by soft gluon radiation.

Like other effective theories NRQCD implies a strict factorization between the short-distance production of the $Q\overline{Q}$ pair and its long-distance fragmentation into the quarkonium. While one can compute the partonic process perturbatively to definite order in the strong coupling constant α_s the non-perturbative physics has to be parameterized into so-called NRQCD matrix elements $\langle \mathcal{O}_c^H(^{2S+1}L_J)\rangle$. They give the probability for the hadronization of a colour singlet/octet ($c = 1/c = 8$) $Q\overline{Q}$ pair with spin S, orbital angular momentum L, and total angular momentum J into the quarkonium H and scale according to power counting rules of NRQCD, i.e. for example that the colour octet matrix elements are suppressed at least by factor of v^2 with respect to the leading order colour singlet contribution.

Although NRQCD matrix elements describe the hadronization adequately in most quarkonium processes, the theoretical prediction of its energy distributions needs more effort: Towards the endpoint of the spectrum the quarkonium takes more and more energy of the incoming particles, i.e. the phase space for radiating off soft gluons from the quark antiquark pair becomes less and less. Therefore one should expect that the colour octet contributions where the $Q\overline{Q}$ pair must emit a soft gluon to get rid of its colour are suppressed for large values of the quarkonium energy. However, in the inelastic J/ψ photoproduction channel naive NRQCD calculations yield a steep raise of the colour octet contributions in the upper endpoint region [2] contradicting the observation of a rather flat spectrum [3]. Thus we parameterize the fragmentation process by functions, the so-called shape functions, rather than by numbers to account for the phase space effect mentioned above.

Technically speaking the necessity of shape functions instead of matrix elements is caused by the breakdown of the NRQCD velocity expansion near the endpoint of quarkonia energy spectra [4]. When the quarkonium carries a fraction $(1-\epsilon)$ of its maximal energy, where ϵ is a small number, the kinematic limitations from the hadronic process appear in the short-distance calculation as expansion in v^2/ϵ. While for small quarkonia energies ($\epsilon \gg v^2$) the naive use of NRQCD is appropriate the endpoint region ($\epsilon \sim v^2$) needs a resummation of the leading twist terms $\mathcal{O}((v/\epsilon)^k)$. Thus one gets a shape function for each production channel separately which smears the partonic spectrum. As result the endpoint moves from the partonic value given by the mass of the $Q\overline{Q}$ pair to the hadronic one set by the quarkonium mass.

The outline of this article is as follows. First we construct a model for the shape function which is consistent with the shape function formalism of [4] and models the region where $\epsilon \ll v^2$. Thereafter we apply our model to the decay $B \to J/\psi X$ and fit the model parameter Λ to the CLEO data. Assuming universality we finally transfer the results from J/ψ production in B decay to the photoproduction channel.[†]

[†]For a more detailed presentation of the contents of this article please refer to [5] by Beneke *et al.*

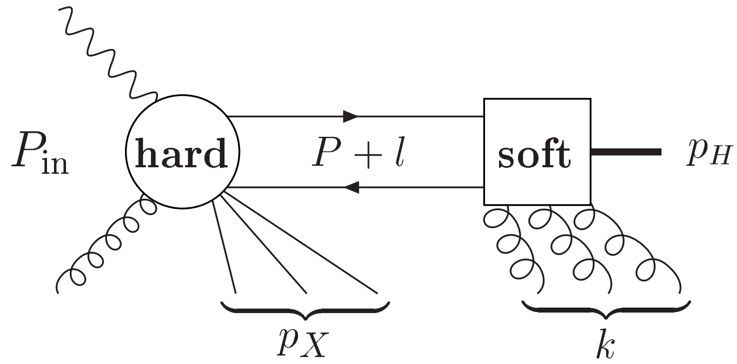

Figure 1: Diagrammatic representation of a general quarkonium production process.

2 Shape function model

Starting point for our analysis is figure 1. Here quarkonium production is considered as a two step process: the short-distance production of the $Q\overline{Q}$ pair described by the partonic cross section $\hat{\sigma}_{Q\overline{Q}}[n]$ and its subsequent fragmentation into the quarkonium H. Hence we get

$$(2\pi)^3\, 2p_H^0 \frac{d\sigma}{d^3p_H} \equiv \sum_n \int \frac{d^4l}{(2\pi)^4}\; \hat{\sigma}_{Q\overline{Q}}[n](l)\cdot F_n(l) \tag{1}$$

where the shape function

$$F_n(l) = \int \frac{dk^2}{2\pi}\frac{d^3\boldsymbol{k}}{(2\pi)^3 2k_0}\;(2\pi)^4\delta^4(P+l-p_H-k)\Phi_n(k;p_H,P) \tag{2}$$

parameterizes the emission of a soft gluon field with total momentum $\boldsymbol{k}$ and invariant mass k^2 from a $Q\overline{Q}$ pair of quantum state $n = {}^{2S+1}L_J^{(c)}$ and with off-shellness $l = p_{Q\overline{Q}} - P$ where $P^2 = 4m_Q^2$. $\Phi_n(k)$ is a radiation function that contains the non-perturbative physics of the hadronization process. Before defining this radiation function we elaborate the phase space exactly.

To this end we restrict ourselves to the case of a single massless hard particle in the final state and refer to the quarkonium rest frame defined by $\boldsymbol{p_H} = \boldsymbol{P} = 0$ rather than to the center-of-mass frame ($\boldsymbol{P}_{\rm in} = 0$). After integrating over the hard momentum p_X and the soft momentum $\boldsymbol{k}$ we switch to light cone variables for l to reproduce the shape function formalism of [4]. We integrate out $l_+ = l_0 + l_z$, $l_\perp^2$ and the azimuthal angular ϕ and rewrite the l_0 integration into a integration over the soft gluon energy k_0. Then the final result reads

$$(2\pi)^3\, 2p_H^0 \frac{d\sigma}{d^3p_H} = \sum_n \int_0^{\alpha\beta} \frac{dk^2}{2\pi} \int_{(\alpha^2+k^2)/(2\alpha)}^{(\beta^2+k^2)/(2\beta)} dk_0 \mathcal{F}\cdot \bar{H}_n(P_{\rm in}, P, l, p_X)\cdot \frac{\Phi_n(k;p_H,P)}{4\pi(\beta-\alpha)} \tag{3}$$

where the integration bounds are functions of

$$\alpha \equiv P_{\rm in+} - M_H\,, \quad \beta \equiv P_{\rm in-} - M_H \tag{4}$$

which again are defined in the quarkonium rest frame.

The hard subprocess $in \to Q\overline{Q}[n] + X$ is given by the flux factor $\mathcal{F}$ and the azimuthal average $\bar{H}_n = \int d\phi/(2\pi) H_n$ of the partonic amplitude squared. While this part can be calculated perturbatively we have to model the radiation function Φ_n. We make the ansatz

$$\Phi_n(k; p_H, P) = a_n \cdot |\boldsymbol{k}|^{b_n} \exp(-k_0^2/\Lambda_n^2) \cdot k^2 \exp(-k^2/\Lambda_n^2) \tag{5}$$

where the exponential cut-off reflects our expectation that the typical energy and invariant mass of the radiated system is of order $\Lambda_n \sim m_Q v^2 \approx$ several hundred MeV for $n = {}^1S_0^{(8)}$, ${}^3P_J^{(8)}$, ${}^3S_1^{(8)}$. Note that there is no soft gluon emission necessary in the colour singlet case, i.e. Φ_n collapses to the delta function $\delta^4(k)$. The normalization a_n is fixed as described in [5]. We choose the other parameters as follows ($c = 1.5$) [5]:

$$b[{}^1S_0^{(8)}] = 2\,, \qquad b[{}^3P_0^{(8)}] = b[{}^3S_1^{(8)}] = 0\,, \tag{6}$$

$$\Lambda[{}^1S_0^{(8)}] = \Lambda[{}^3P_0^{(8)}] \equiv \Lambda\,, \quad \Lambda[{}^3S_1^{(8)}] = c\Lambda\,. \tag{7}$$

3 Momentum spectrum in $B \to J/\psi X$

Let us now apply the formalism to the J/ψ momentum spectrum in the semi-inclusive decay $B \to J/\psi X$. The leading partonic decay process $b \to c\bar{c}[n] + q$ ($q = \{d, s\}$) results in a J/ψ with fixed momentum but the hadronic decay spectrum is modified by both initial and final bound state effects. The Fermi motion of the b quark inside the B meson is accounted by the ACCMM model in a simple but satisfactory way. The fragmentation effects are considered by our shape function formalism. Colour octet contributions are non-negligible because their suppression due to a scaling factor of $v^4 \approx 1/15$ from the NRQCD matrix elements is compensated by a factor $C_{[8]}^2/C_{[1]}^2 \approx 15$ from the Wilson coefficients in the partonic process.

Thus we take the partonic amplitude squared $H_n(m_b, 2m_c)$ for $n = {}^1S_0^{(8)}$, ${}^3P_J^{(8)}$, ${}^3S_1^{(8)}$, and ${}^3S_1^{(1)}$ at tree level [6] and perform the substitution

$$2m_c \to M_{c\bar{c}}(k) = \sqrt{M_{J/\psi}^2 + 2M_{J/\psi}k_0 + k^2} \tag{8}$$

to allow the $c\bar{c}$ pair being off-shell before it emits soft gluons. Then the J/ψ energy distribution in b quark decay is obtained by folding the partonic result with our shape function model. Finally we use this result as input for the ACCMM model to include initial bound state effects. It assumes an isotropic Fermi motion of the b quark inside the meson and a Gaussian momentum distribution with a width p_F of several hundreds MeV. Furthermore the ACCMM model keeps the kinematics of the b quark decay 'in flight' exact treating the soft degrees of freedom inside the B meson as spectator of mass m_{sp}.

CLEO has measured the J/ψ momentum spectrum on the $\Upsilon(4S)$ resonance [7]. Hence we boost our result from the B meson rest frame to their laboratory system. We then assume that the colour singlet contribution is dual to the exclusive modes $B \to J/\psi K^{(*)}$ [5].

The result for the branching ratio with $J/\psi K^{(*)}$ subtracted is shown in figure 2. It is clearly seen that the effect of $c\bar{c}$ fragmentation is necessary to reproduce the data for an ordinary choice for the ACCMM parameters ($p_F = 300$ MeV, $m_{sp} = 150$ MeV). In other words $\Lambda = 0$ cannot describe the data without pushing p_F up to values far above the ones

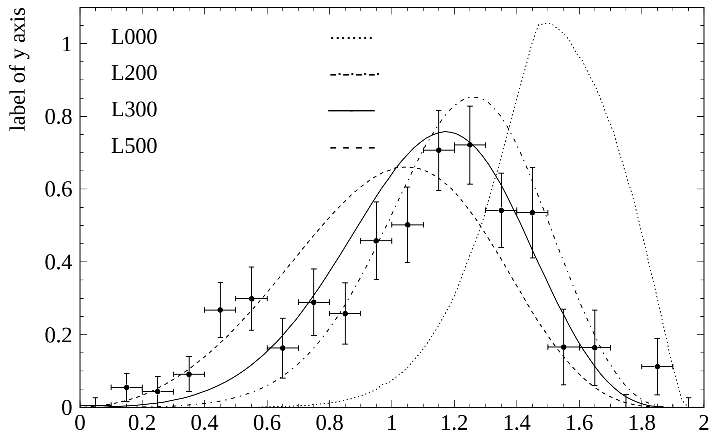

Figure 2: Sum of colour octet modes $d\mathrm{BR}_{[8]}/dp_R$ to the differential branching ratio of the decay $B \to J/\psi X$ compared to CLEO data [7].

successfully employed e.g. in semi-leptonic B decays. If we fit the shape function model parameter Λ we obtain

$$\Lambda = 300^{+50}_{-100}\,\mathrm{MeV} \tag{9}$$

which excellently fits to our expectation $\Lambda \sim m_c v^2 \sim \Lambda_{\mathrm{QCD}}$ from the NRQCD velocity scaling rules. The uncertainties in (9) come from a different weighting between the colour octet contributions and from varying p_F from 200 MeV to 500 MeV.

4 Inelastic J/ψ photoproduction

In this section we discuss the energy spectrum in inelastic J/ψ photoproduction. Again we take the squared amplitude H_n from the partonic calculation [2] and perform the substitution (8) $2m_c \to M_{c\bar{c}}(k)$.[‡] In this case it is more complicated to incorporate the shape function model (3) since the incoming particles distinguish a symmetry axis in the partonic process $g+\gamma \to c\bar{c}[n]+g$. We will keep exact its perpendicular $l_\perp$ dependence even though it is of higher order in velocity scaling. Next we employ the parton density function to include initial state effect and integrate over the proton momentum fraction of the gluon and the transverse momentum p_T of the J/ψ to get its energy spectrum $d\sigma(\gamma p \to J/\psi X)/dz$ where $z = (p_{J/\psi}\cdot p_p)/(p_\gamma\cdot p_p)$ is the photon energy fraction transferred to the J/ψ.

Finally a comment on the normalization of our result. The relative weights of the different production channels are set by the corresponding values of the NRQCD matrix elements [5]. The overall normalization is fixed by adjusting the shape function modified curvature to the one from naive NRQCD in the region $0.1 \leq z \leq 0.4$. Thus we account for the effect that partonically the c quark mass (8) is higher than the value $m_c = 1.5$ GeV used in the fits for the NRQCD matrix elements.

Figure 3 displays the result for $p_T \geq 1$ GeV and $p_T \geq 2$ GeV plotted against data from HERA [3]. Due to our normalization procedure the shape function improved curves

[‡]There is a misprint in the original paper [5]: In each mode of H_n the factor g_s^2 should be substituted by g_s^4. The numerics, however, remain unchanged.

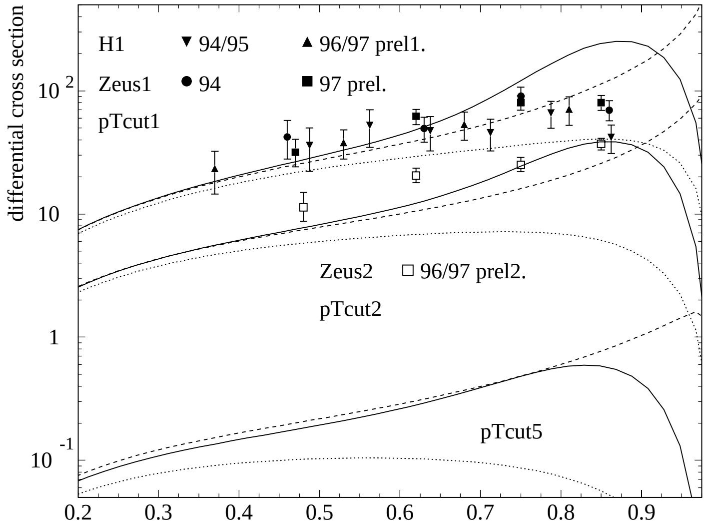

Figure 3: J/ψ energy spectrum in photoproduction for different values of the transverse momentum cut ($p_T^{\rm cut} = 1, 2, 5$ GeV): colour singlet model (dotted), naive (dashed) and shape function modified (solid) NRQCD calculation versus HERA data for $p_T \geq 1$ GeV and ≥ 2 GeV respectively.

coincide with the naive NRQCD calculation for low values of z. However, they display a strong enhancement in the region of intermediate z before they fall down to zero for $z \to 1$ as anticipated from general arguments.

The explanation for this (unphysical) strong enhancement lies in the structure of partonic colour octet amplitudes. Since H_n is proportional to $(1 - z_{c\bar{c}})^{-2}$ for $n = {}^1S_0^{(8)}$ and $n = {}^3P_J^{(8)}$ where $z_{c\bar{c}}$ is the corresponding quantity to z for the $c\bar{c}$ pair we get large contributions from $z_{c\bar{c}} \to 1$. Unfortunately soft gluon radiation in the final state decouples the energy (as well as the transverse momentum) of the J/ψ and the $c\bar{c}$ pair, i.e. including our model we sample the partonic rate for $z_{c\bar{c}} \geq z$ close to one even though the cut on p_T sets an upper bound on z. To fix this problem we repeated the analysis with a higher p_T cut (cp. figure 3). Now the hunch disappears because $z^{\rm max}$ is small enough to keep $z_{c\bar{c}}$ away from one.

In conclusion one should note that it is impossible to neglect or even rule out colour octet contributions in the photoproduction channel from the observation of a flat energy spectrum without increasing $p_T^{\rm cut}$.

Acknowledgements

I would like to thank M. Beneke for useful comments and careful reading the manuscript. The author is supported by the Graduiertenkolleg “Elementarteilchenphysik an Beschleunigern” and the DFG-Forschergruppe “Quantenfeldtheorie, Computeralgebra und Monte-Carlo-Simulation”.

References

[1] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51** (1995) 1125, Erratum: Phys. Rev. **D55** (1997) 5853.

[2] M. Cacciari and M. Krämer, Phys. Rev. Lett. **76** (1996) 4128.

[3] S. Aid *et al.* [H1 Collaboration], Nucl. Phys. **B472** (1996) 3. K. Krüger, private communication. J. Breitweg *et al.* [ZEUS Collaboration], Z. Phys. **C76** (1997) 599. A. Bertolin, private communication and Proceedings of ICHEP2000, Osaka, Japan, August 2000.

[4] M. Beneke, I.Z. Rothstein and M.B. Wise, Phys. Lett. **B408** (1997) 373.

[5] M. Beneke, G. A. Schuler and S. Wolf, Phys. Rev. **D62** (2000) 034004.

[6] P. Ko, J. Lee and H. S. Song, Phys. Rev. **D53** (1996) 1409.

[7] R. Balest *et al.* [CLEO Collaboration], Phys. Rev. **D52** (1995) 2661.